Terminating combinator parsers in Agda

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Overview

Totality

Parser combinators

Terminating combinator parsers
Totality
Total functions

A function is called **total** if it terminates and produces a valid (non-⊥) result for any input.

```
Many Haskell functions are not total:

head :: [a] → a
head (x:xs) = x  
Fails on the empty list.

factorial :: Int → Int
factorial 0 = 1
factorial n = n * factorial (n - 1)  
Loops on any negative input.
```
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Why?
Reasons for totality

- In Haskell, every type is **inhabited**: ⊥ is a value of any type.
Reasons for totality

- In Haskell, every type is **inhabited**: \( \bot \) is a value of any type.
- In dependently typed languages, we want to use the Curry-Howard correspondence: types are propositions, values are proofs.

```haskell
data _≤_ : ℕ → ℕ → Set where
  ≤ base : ∀{n} → n ≤ n
  ≤ step : ∀{m n} → m ≤ n → m ≤ suc n
  trans : ∀{l m n} → l ≤ m → m ≤ n → l ≤ n
  replaceInits : ∀{a m n} → m ≤ n → Vec a m → Vec a n → Vec a n
```
Reasons for totality

- In Haskell, every type is **inhabited**: \( \bot \) is a value of any type.
- In dependently typed languages, we want to use the Curry-Howard correspondence: types are propositions, values are proofs.

```haskell
data _ <= _ : \mathbb{N} \to \mathbb{N} \to \text{Set} where
  <= base : \forall \{n\} \rightarrow n \leq n
  <= step : \forall \{m n\} \rightarrow m \leq n \rightarrow m \leq \text{suc} n
  trans : \forall \{l m n\} \rightarrow l \leq m \rightarrow m \leq n \rightarrow l \leq n
  replaceInits : \forall \{a m n\} \rightarrow m \leq n \rightarrow
    \text{Vec} a m \rightarrow \text{Vec} a n \rightarrow \text{Vec} a n
```

- Haskell is **inconsistent**: all propositions can be proved.
Reasons for totality – contd.

▶ Types can contain terms:

\[
\text{Vec : Set} \to \mathbb{N} \to \text{Set} \\
_+ : \forall\{a \ m \ n\} \to \text{Vec} \ a \ m \to \text{Vec} \ a \ n \to \text{Vec} \ a \ (m + n) \\
\text{tail} : \forall\{a \ n\} \to \text{Vec} \ a \ (\text{suc} \ n) \to \text{Vec} \ a \ n
\]
Reasons for totality – contd.

- Types can contain terms:

  \[
  \text{Vec : Set} \rightarrow \mathbb{N} \rightarrow \text{Set} \\
  _+ + _ : \forall \{a \ m \ n\} \rightarrow \text{Vec } a \ m \rightarrow \text{Vec } a \ n \rightarrow \text{Vec } a \ (m + n) \\
  \text{tail} : \forall \{a \ n\} \rightarrow \text{Vec } a \ (\text{suc } n) \rightarrow \text{Vec } a \ n
  \]

- Consider:

  \[
  \text{tail } (v_1 + v_2)
  \]
 Reasons for totality – contd.

- Types can contain terms:

  \[
  \text{Vec} : \text{Set} \rightarrow \mathbb{N} \rightarrow \text{Set} \\
  \text{Vec a m} \rightarrow \text{Vec a n} \rightarrow \text{Vec a} (m + n) \\
  \text{tail} : \forall \{a \ n\} \rightarrow \text{Vec a} \ (\text{suc n}) \rightarrow \text{Vec a} \ n
  \]

- Consider:

  \text{tail} (v_1 \oplus v_2)

- Typechecking the expression requires unification:

  \((\text{length v}_1 + \text{length v}_2) \sim \text{suc n})

  (for any n).
Consequences of totality

- Inductively defined datatypes have only finite values.
- Evaluation strategy (eager vs. lazy) is semantically irrelevant.
- The language cannot be Turing-complete (but still surprisingly expressive).
How to ensure totality

▶ Agda has a built-in coverage and termination checker.
▶ The coverage checker ensures that in a case analysis, all possible patterns are covered.
▶ The termination checker essentially checks that functions are structurally recursive.
Structural recursion

- Each value essentially is a constructor applied to other values:

  \[ v = C \, v_1 \ldots v_n \]

- All such subvalues (and their subvalues . . . ) are structurally smaller. Recursive calls must make at least one argument structurally smaller.

Some functions are trivially structurally recursive:

- length: \( \forall \{ a \} \to [a] \to \mathbb{N} \)

  \[ \text{length} \, [\,] = 0 \]

  \[ \text{length} \, (x :: xs) = 1 + \text{length} \, xs \]
Structural recursion

- Each value essentially is a constructor applied to other values:

\[ v = C \, v_1 \ldots v_n \]

- All such subvalues (and their subvalues . . .) are structurally smaller. Recursive calls must make at least one argument structurally smaller.

- Many functions are trivially structurally recursive:

\[
\begin{align*}
\text{length} & : \forall\{a\} \to [a] \to \mathbb{N} \\
\text{length} & [\ ] = 0 \\
\text{length} & (x :: xs) = 1 + \text{length} \, xs
\end{align*}
\]

Others (e.g. Quicksort) require some work . . .
Parser combinators
Simple parsers

We can do better, but for this talk, we choose a naïve implementation (list of successes):

\[
\text{Input} : \text{Set} \\
\text{Input} = [\text{Char}] \\
\text{Parser} : \text{Set} \rightarrow \text{Set} \\
\text{Parser } r = \text{Input} \rightarrow [r \times \text{Input}]
\]
Applicative interface

fail : \(\forall \{ r \} \to \text{Parser } r\)
fail \(\text{inp} = []\)

succeed : \(\forall \{ r \} \to r \to \text{Parser } r\)
succeed \(x \text{ inp} = (x, \text{inp}) :: []\)
Applicative interface

fail : \( \forall\{ r \} \rightarrow \text{Parser } r \)
fail inp = []

succeed : \( \forall\{ r \} \rightarrow r \rightarrow \text{Parser } r \)
succeed x inp = (x, inp) :: []

_ \_ \_ : \( \forall\{ r \} \rightarrow \text{Parser } r \rightarrow \text{Parser } r \rightarrow \text{Parser } r \)
(p \_ \_ \_ q) inp = p inp ++ q inp
**Applicative interface**

\[
\begin{align*}
\text{fail} & : \forall \{ r \} \rightarrow \text{Parser } r \\
\text{fail} \ \text{inp} & = [] \\
\text{succeed} & : \forall \{ r \} \rightarrow r \rightarrow \text{Parser } r \\
\text{succeed} \ x \ \text{inp} & = (x, \ \text{inp}) :: [] \\
\_ \ & \_ : \forall \{ r \} \rightarrow \text{Parser } r \rightarrow \text{Parser } r \rightarrow \text{Parser } r \\
(p \ \_ \ q) \ \text{inp} & = p \ \text{inp} \ \_ \ q \ \text{inp} \\
\& \ _ : \forall \{ r \ s \} \rightarrow \text{Parser } (r \rightarrow s) \rightarrow \text{Parser } r \rightarrow \text{Parser } s \\
(p \ \& \ q) \ \text{inp} & = \\
& \text{concat} \ (\text{map} \ (\lambda f \rightarrow \text{map} \ (\lambda g \rightarrow ((\pi_1 f) \ (\pi_1 g), \pi_2 g)) \ q \ (\pi_2 f))) \ (p \ \text{inp}))
\end{align*}
\]
Applicative interface – contd.

symbol : Char → Parser Char
symbol _ [] = []
symbol x (i :: inp) = if i == x then [x, inp] else []

($) _ : ∀{r s} → (r → s) → Parser r → Parser s
f $ p = succeed f ∗ p
Applicative interface – contd.

symbol : Char → Parser Char
symbol _ [] = []
symbol x (i :: inp) = if i == x then [x, inp] else []
_ $ _ : ∀{r s} → (r → s) → Parser r → Parser s
f $ p = succeed f * p

- The combinators are not recursive and thus accepted as total functions by Agda.
Applicative interface – contd.

symbol : Char \rightarrow Parser Char
symbol \_ [] = []
symbol \times (i :: inp) = if i == x then [x, inp] else []

\_ $ \_ : \forall \{r s\} \rightarrow (r \rightarrow s) \rightarrow Parser r \rightarrow Parser s
f $ p = succeed f \star p

- The combinators are not recursive and thus accepted as total functions by Agda.
- However, nearly all interesting grammars are cyclic, and the resulting combinator parsers recursive:

sum : Parser \mathbb{N}
sum = (\lambda m \_ n \rightarrow m + n) \$ nat \star symbol ' + ' \star sum
\_ nat
Not all parsers terminate

\[
\text{nat} : \text{Parser } \mathbb{N} \\
\text{nat} = (\lambda n \text{ d} \rightarrow n \times 10 + \text{d}) \$ \text{nat } \star \text{digit} \\
\star \text{digit}
\]
Not all parsers terminate

\[\text{nat} : \text{Parser } \mathbb{N} \]
\[\text{nat} = (\lambda n d \to n \times 10 + d) \ $ \text{nat} \star \text{digit} \]
\[\ | \ \text{digit} \]

\[\text{many} : \forall\{a\} \to \text{Parser } a \to \text{Parser } [a] \]
\[\text{many } p = _:: _ \ $ p \star \text{many } p \]
\[\ | \ \text{succeed } [] \]

\[\text{optx} : \text{Parser Char} \]
\[\text{optx} = \text{symbol } 'x' \ | \ \text{succeed } ' ' \]

\[\text{optxs} : \text{Parser } [\text{Char}] \]
\[\text{optxs} = \text{many } \text{optx} \]
The rest of this talk

- We will design parser combinators so that the resulting parsers are structurally recursive.
- Left-recursive grammars (directly and indirectly) will be type-incorrect in this library.
Terminating combinator parsers
The main idea

- Look at the following graph: nodes are parsers, an edge from one node to another indicates that a parser can directly call another (without first consuming a symbol).
- For left-recursive grammars (directly or indirectly), the graph contains cycles.
- For other grammars, the graph is cycle-free, and can be expanded into a finite tree.
- If we make this tree an index of the parser type, then left-recursive parsers are no longer type-correct.
Dependency tree

data Corners : Set where
  leaf : Corners
  node_1 : Corners → Corners
  node_2 : Corners → Corners → Corners
Parser : Corners → Set → Set
data Corners : Set where
  leaf : Corners
  node₁ : Corners → Corners
  node₂ : Corners → Corners → Corners
Parser : Corners → Set → Set

symbol : Char → Parser leaf Char
succeed : ∀{r} → r → Parser leaf r
_ · _ : ∀{r} → Parser c₁ r → Parser c₂ r →
  Parser (node₂ c₁ c₂) r
_ ∗ _ : ∀{r s} → Parser c₁ (r → s) → Parser c₂ r →
  Parser ? s
It is important to know if a parser accepts the empty word:

Empty : Set
Empty = Bool
Parser : (Empty × Corners) → Set → Set
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Empty : Set
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symbol : Char → Parser (false, leaf) Char
Epsilon

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Empty : Set
Empty = Bool
Parser : (Empty × Corners) → Set → Set

symbol : Char → Parser (false, leaf) Char
succeed : ∀{ r } → r → Parser (true, leaf) r
Epsilon

It is important to know if a parser accepts the empty word:

Empty : Set
Empty = Bool
Parser : (Empty × Corners) → Set → Set

symbol : Char → Parser (false, leaf) Char
succeed : ∀{ r } → r → Parser (true, leaf) r
_not_ : ∀{ e_1 c_1 e_2 c_2 r } →
    Parser (e_1, c_1) r → Parser (e_1, c_2) r →
    Parser (e_1 ∨ e_2, node_2 c_1 c_2) r
It is important to know if a parser accepts the empty word:

\[
\text{Empty} : \text{Set} \\
\text{Empty} = \text{Bool} \\
\text{Parser} : (\text{Empty} \times \text{Corners}) \rightarrow \text{Set} \rightarrow \text{Set}
\]

\[
\text{symbol} : \text{Char} \rightarrow \text{Parser} (\text{false}, \text{leaf}) \text{ Char} \\
\text{succeed} : \forall\{ r \} \rightarrow r \rightarrow \text{Parser} (\text{true}, \text{leaf}) r \\
\_ \_ _ \_ : \forall\{ e_1 \ c_1 \ e_2 \ c_2 \ r \} \rightarrow \\
\qquad \text{Parser} (e_1, c_1) r \rightarrow \text{Parser} (e_1, c_2) r \rightarrow \\
\qquad \text{Parser} (e_1 \lor e_2, \text{node}_2 \ c_1 \ c_2) r \\
\_ \_ * _ : \forall\{ e_1 \ c_1 \ e_2 \ c_2 \ r \ s \} \rightarrow \\
\qquad \text{Parser} (e_1, c_1) (r \rightarrow s) \rightarrow \text{Parser} (e_1, c_2) r \rightarrow \\
\qquad \text{Parser} (e_1 \land e_2, \text{if } e_1 \text{ then } \text{node}_2 \ c_1 \ c_2 \ \text{else } c_1) s
\]
What about

\[ \text{Parser} : (\text{Empty} \times \text{Corners}) \rightarrow \text{Set} \rightarrow \text{Set} \]

If, as before

\[ \text{Parser} \_ r = \text{Input} \rightarrow [r \times \text{Input}] \]

then the index information is lost!
What about

Parser : (Empty × Corners) → Set → Set

If, as before

Parser ∈ r = Input → [r × Input]

then the index information is lost!

We have to turn Parser into an abstract datatype:

data Parser : (Empty × Corners) → Set → Set where

...
Not done – contd.

- Recursive definitions still pose a problem.
- Does not pass the termination checker, but still type-correct:

  \[ p : \text{Parser} \ (\text{true}, \text{leaf}) \ \text{Char} \]
  \[ p = p \]
Not done – contd.

- Recursive definitions still pose a problem.
- Does not pass the termination checker, but still type-correct:

\[
\begin{align*}
\text{p} : \text{Parser} \ (\text{true}, \text{leaf}) \ \text{Char} \\
\text{p} = \text{p}
\end{align*}
\]

- Recursion must change the Corners tree!

\[
\begin{align*}
! : \forall \{e \ c \ r\} \rightarrow \\
\text{Parser} \ (e, c) \ r \rightarrow \text{Parser} \ (e, \text{node}_1 \ c) \ r
\end{align*}
\]

Recursion via \(!\) fails the “occurs check”:

\[
\begin{align*}
\text{p} = \! \text{p}
\end{align*}
\]
Legal cyclic definitions are still far from structurally recursive:

\[
p : \text{Parser} \\
p = \ldots p \ldots
\]

Turn parsers (thus Corners) into function arguments. This unfortunately has quite a few implications: we turn parser combinators and also the nonterminals of grammars into datatypes, so that we can perform case analysis in a function.
Abstract parsers

ParserType = (Empty \times Corners) \rightarrow Set \rightarrow Set_1

data Parser (nt : ParserType) : ParserType where

! _ : \forall \{e c r\} \rightarrow
    nt (e, c) r \rightarrow Parser nt (e, node_1 c) r

symbol : Char \rightarrow Parser nt (false, leaf) Char

return : \forall \{r\} \rightarrow r \rightarrow Parser nt (true, leaf) r

...
Grammars

Grammar : ParserType → Set₁
Grammar nt = ∀{e c r} → nt (e, c) r → Parser nt (e, c) r

data NT : ParserType where
  nat : NT (_, _) ℕ  -- indices can be inferred!
  sum : NT (_, _) ℕ

grammar : Grammar NT
grammar nat = (const 1) $ sym ’1’  -- simplified
grammar sum = (λm _ n → m + n) $ !nat ∗ symbol ’+’ ∗ !sum
               ∴ !nat
Grammars

Grammar : ParserType → Set₁
Grammar nt = ∀{e c r} → nt (e, c) r → Parser nt (e, c) r

data NT : ParserType where
  nat : NT (__, __) ℕ  -- indices can be inferred!
  sum : NT (__, __) ℕ
grammar : Grammar NT
grammar nat  = (const 1) $ sym ’1’  -- simplified
grammar sum  = (λm _ n → m + n) $ !nat ∗ symbol ’+’ ∗ !sum
       ∣  !nat

The definition of grammar is type correct if no left-recursion is involved. It is no longer recursive.
Interpreting the parsers

```
parse : { nt : ParserType } (g : Grammar nt)
    { e : Empty } { c : Corners } { r : Set } →
   Parser nt (e, c) r →
LoS.Parser r   -- original parser type

parse g (!p)         = parse g (g p)
parse g (symbol c)   = LoS.symbol c
parse g (p₁ ∪ p₂)    = LoS._ ∪ _ (parse g p₁) (parse g p₂)
parse g (p₁ * p₂)    = LoS._ * _ (parse g p₁) (parse g p₂)
```

Is this definition structurally recursive?
No, in the ∪ case, the structure of the parser can get larger; in the ⋆ case, p₂ can have a large Corners tree.
Interpreting the parsers

\[
\text{parse} : \{ \text{nt} : \text{ParserType} \}(g : \text{Grammar nt})
\{ \text{e} : \text{Empty} \}\{ \text{c} : \text{Corners} \}\{ \text{r} : \text{Set} \} \rightarrow
\text{Parser nt (e, c) r} \rightarrow
\text{LoS.Parser r} \quad \text{-- original parser type}
\]

\[
\begin{align*}
\text{parse g (p)} & = \text{parse g (g p)} \\
\text{parse g (symbol c)} & = \text{LoS.symbol c} \\
\text{parse g (p₁ | p₂)} & = \text{LoS._ | _ (parse g p₁) (parse g p₂)} \\
\text{parse g (p₁ ⋄ p₂)} & = \text{LoS._ ⋄ _ (parse g p₁) (parse g p₂)}
\end{align*}
\]

Is this definition structurally recursive?
Interpreting the parsers

\[
\text{parse} : \{ \text{nt} : \text{ParserType} \}(g : \text{Grammar } \text{nt}) \\
\{ e : \text{Empty} \}\{ c : \text{Corners} \}\{ r : \text{Set} \} \rightarrow \\
\text{Parser } \text{nt} \{ e, c \} \text{ r } \rightarrow \\
\text{LoS.} \text{Parser } \text{r} \quad -- \text{original parser type}
\]

\[
\begin{align*}
\text{parse } g (!p) &= \text{parse } g (g \ p) \\
\text{parse } g (\text{symbol } c) &= \text{LoS.} \text{symbol } c \\
\text{parse } g (p_1 \mid p_2) &= \text{LoS.} \_ \mid \_ \text{ (parse } g \ p_1 \text{ ) (parse } g \ p_2 \text{ )} \\
\text{parse } g (p_1 \star p_2) &= \text{LoS.} \_ \star \_ \text{ (parse } g \ p_1 \text{ ) (parse } g \ p_2 \text{ )}
\end{align*}
\]

Is this definition structurally recursive?
Interpreting the parsers

\[
\text{parse} : \{ \text{nt} : \text{ParserType}\}(g : \text{Grammar nt})
\{\text{e} : \text{Empty}\}\{\text{c} : \text{Corners}\}\{\text{r} : \text{Set}\} \rightarrow
\text{Parser nt (e, c) r} \rightarrow
\text{LoS.Parser r} \quad -- \text{original parser type}
\]

\[
\text{parse g (!p)} = \text{parse g (g p)}
\]

\[
\text{parse g (symbol c)} = \text{LoS.symbol c}
\]

\[
\text{parse g (p}_1 \mid \text{p}_2) = \text{LoS.} \_ \mid \_ (\text{parse g p}_1) (\text{parse g p}_2)
\]

\[
\text{parse g (p}_1 \ast \text{p}_2) = \text{LoS.} \_ \ast \_ (\text{parse g p}_1) (\text{parse g p}_2)
\]

Is this definition structurally recursive?

No, in the ! case, the structure of the parser can get larger; in the \( \ast \) case, \( p_2 \) can have a large Corners tree.
A final refinement

We refine the Input type to keep an upper bound of the length of the input string:

\[
\text{Input} : \mathbb{N} \rightarrow \text{Set} \\
\text{Input } n = \text{BoundedVec Char } n \\
\text{Parser} : \mathbb{N} \rightarrow \mathbb{N} \rightarrow \text{Set} \rightarrow \text{Set} \\
\text{Parser } m \ n \ r = \text{Input } m \rightarrow [r \times \text{Input } n]
\]
Adapting parse

parse : \{ nt : ParserType \}(g : Grammar nt) 
(n : \mathbb{N})\{ e : Empty \}\{ c : Corners \}\{ r : Set \} \rightarrow 
Parser nt (e, c) r \rightarrow 
LoS Parser n (if e then n else pred n) r

...
Adapting parse

\[
\text{parse} : \{ \text{nt : ParserType}\} (g : \text{Grammar nt}) \ (n : \mathbb{N}) \{ e : \text{Empty}\} \{ c : \text{Corners}\} \{ r : \text{Set}\} \rightarrow \\
\text{Parser nt (e, c) r } \rightarrow \\
\text{LoS.Parser n (if e then n else pred n) r}
\]

\[
\ldots
\]

\[
\text{parse g n (} (_* _ (e_1 = \text{true}) p_1 p_2) = \text{LoS.} _ * _ (\text{parse g n p}_1) (\text{parse g n p}_2)
\]

-- ok because \(p_1\) and \(p_2\) have a smaller Corners tree
Adapting parse

\[
\text{parse} : \{ nt : \text{ParserType} \}(g : \text{Grammar nt}) \\\n(n : \mathbb{N})\{ e : \text{Empty} \}\{ c : \text{Corners} \}\{ r : \text{Set} \} \rightarrow \\\n\text{Parser } nt \ (e, c) \ r \rightarrow \\\n\text{LoS.}\text{Parser } n \ (\text{if } e \text{ then } n \text{ else } \text{pred } n) \ r
\]

\[
\ldots \text{parse } g \ n \ (\_ * \_ \{ e_1 = \text{true} \} \ p_1 \ p_2) \\\n= \text{LoS.}\_ * \_ (\text{parse } g \ n \ p_1) \ (\text{parse } g \ n \ p_2) \\\n\quad -- \text{ok because } p_1 \text{ and } p_2 \text{ have a smaller Corners tree} \\\n\text{parse } g \ 0 \ (\_ * \_ \{ e_1 = \text{false} \} \ p_1 \ p_2) \\\n= \text{LoS.}\text{fail}
\]
Adapting parse

\[
\text{parse} : \{ nt : \text{ParserType} \} (g : \text{Grammar nt}) \\
(n : \mathbb{N}) \{ e : \text{Empty} \} \{ c : \text{Corners} \} \{ r : \text{Set} \} \rightarrow \\
\text{Parser nt} (e, c) r \rightarrow \\
\text{LoS.Parser n} (\text{if } e \text{ then } n \text{ else } \text{pred n}) r
\]

\[
\ldots
\]

\[
\text{parse } g \ n \quad (\_ \star \_ \{ e_1 = \text{true} \} p_1 \ p_2) \\
= \text{LoS.} \_ \star \_ (\text{parse } g \ n \ p_1) \ (\text{parse } g \ n \ p_2)
\]

\[
\quad \text{-- ok because } p_1 \text{ and } p_2 \text{ have a smaller Corners tree}
\]

\[
\text{parse } g \ 0 \quad (\_ \star \_ \{ e_1 = \text{false} \} p_1 \ p_2)
\]

\[
= \text{LoS.fail}
\]

\[
\text{parse } g \ (\text{suc n}) \ (\_ \star \_ \{ e_1 = \text{false} \} p_1 \ p_2)
\]

\[
= \text{LoS.} \_ \star \_ (\text{parse } g \ (\text{suc n}) \ p_1) \ (\text{parse}^{\uparrow} g \ n \ p_2)
\]

\[
\text{parse}^{\uparrow} : \ldots \rightarrow \\
\quad \text{-- like parse, but results in } \ldots
\]

\[
\text{LoS.Parser } n \ n \ r
\]
Summary

▶ We have shown that structurally recursive parser combinators can be implemented in Agda.
▶ Parsers written using this library are total. Left-recursive grammars (whether directly or indirectly) are rejected at compilation time.
▶ More work for the implementor, not much more work for the user, except . . .
▶ Defining reusable recursive derived combinators (e.g. many) requires a bit of additional trickery.
▶ The indices (Empty and Corners) can usually be inferred.
▶ Efficiency in current implementations is not too good, but in principle, not much overhead is involved – most of the indices are irrelevant at run time and can be eliminated.
Interested in Agda?

Try the seminar on

“Dependently Typed Programming”
(INFOMDTP)