Structural polymorphism in Generic Haskell

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Genericity and other types of polymorphism

Examples of generic functions

Generic Haskell
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Genericity and other types of polymorphism

Examples of generic functions

Generic Haskell
Haskell

- Haskell is a statically typed, pure functional language with lazy evaluation.

- Functions are defined by pattern matching.

  \[
  \text{factorial} \ 0 \ = \ 1 \\
  \text{factorial} \ n \ = \ n \cdot \text{factorial} \ (n - 1)
  \]

- Every function has a type that usually can be inferred by the compiler.

  \[
  \text{factorial} :: \text{Int} \rightarrow \text{Int}
  \]

- Functions with multiple arguments are written in curried style.

  \[
  \text{and} :: \text{Bool} \rightarrow \text{Bool} \rightarrow \text{Bool} \\
  \text{and} \ True \ True = \ True \\
  \text{and} \ _ \ _ = \ False
  \]
Haskell

- Haskell is a **statically typed** language.
- Functions are defined by pattern matching.

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\begin{align*}
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\end{align*}
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- Every function has a type that usually can be **inferred** by the compiler.

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\text{factorial} :: \text{Int} \to \text{Int}
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- Functions with multiple arguments are written in **curried** style.

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Haskell

- Haskell is a **statically typed** language.
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  ```haskell
  factorial 0 = 1
  factorial n = n * factorial (n - 1)
  ```
- Every function has a type that usually can be **inferred** by the compiler.
  
  ```haskell
  factorial :: Int -> Int
  ```
- Functions with multiple arguments are written in **curried** style.
  
  ```haskell
  and :: Bool -> Bool -> Bool
  and True True = True
  and _ _ = False
  ```
New datatypes can be defined in Haskell using the `data` construct:

```haskell
data Nat = Zero | Succ Nat
```

The expression `Succ (Succ (Succ Zero))` represents the number 3.

Functions are often defined recursively, by induction on the structure of a datatype:

```haskell
plus :: Nat → Nat → Nat
plus m Zero = m
plus m (Succ n) = Succ (plus m n)
```
New **datatypes** can be defined in Haskell using the **data** construct:

```
data Nat = Zero | Succ Nat
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The expression \( \text{Succ} \ (\text{Succ} \ (\text{Succ} \ Zero)) \) represents the number 3.

Functions are often defined recursively, by **induction on the structure** of a datatype:

```
plus :: Nat → Nat → Nat
plus m Zero = m
plus m (Succ n) = Succ (plus m n)
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Haskell datatypes

Haskell’s **data** construct is extremely flexible.

```haskell
data TimInfo = AM | PM | H24
data Package = P String Author Version Date
data Maybe α = Nothing | Just α
data [α] = [] | α : [α]
data Tree α = Leaf α | Node (Tree α) (Tree α)
```

Common structure:
- parametrized over a number of arguments
- several *constructors* / alternatives
- multiple *fields* per constructor
- possibly *recursion*
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Generic Haskell
Haskell allows to express functions that work on all datatypes in a uniform way.

\[ id :: \forall \alpha. \alpha \to \alpha \]
\[ id \; x = x \]

\[ swap :: \forall \alpha \; \beta. (\alpha, \beta) \to (\beta, \alpha) \]
\[ swap \; (x, y) = (y, x) \]

\[ head :: \forall \alpha. [\alpha] \to \alpha \]
\[ head \; (x : xs) = x \]

We can take the head of a list of Packages, or swap a tuple of two Trees.
When are two values equal?

It is easy to define an equality function for a specific datatype.

- Both values must belong to the same alternative.
- The corresponding fields must be equal.
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- Both values must belong to the same alternative.
- The corresponding fields must be equal.
They must belong to the same alternative

\begin{verbatim}
data TimelInfo = AM | PM | H24

(==) TimelInfo :: TimelInfo → TimelInfo → Bool

AM == TimelInfo AM = True
PM == TimelInfo PM = True
H24 == TimelInfo H24 = True
– == TimelInfo – = False
\end{verbatim}
The corresponding fields must be equal

```haskell
data Package = PD String Author Version Date

(==) :: Package → Package → Bool

(PD n c v d) == (PD n' c' v' d') =
  n == (String n')
  ∧ c == (Author c')
  ∧ v == (Version v')
  ∧ d == (Date d')
```
Equality for parametrized datatypes

```haskell
data Maybe α = Nothing | Just α

(=·) Maybe :: ∀α. (α → α → Bool) → (Maybe α → Maybe α → Bool)

(=·) Maybe Nothing Nothing = True

(=·) Maybe (Just x) (Just x') = x =?= x'

(=·) Maybe _ _ = False
```

- We can define the equality for parametrized datatypes, but for that, we must know the equality function(s) for the argument(s).
- The equality function depends on itself.
Equality for parametrized datatypes

```latex
\textbf{data} \text{Maybe} \alpha = \text{Nothing} \mid \text{Just} \alpha

(\mathit{=\!\!\!\!\_})_{\text{Maybe}} :: \forall \alpha. (\alpha \to \alpha \to \text{Bool}) \to (\text{Maybe} \alpha \to \text{Maybe} \alpha \to \text{Bool})

(\mathit{=\!\!\!\!\_})_{\text{Maybe}} (\mathit{=\!\!\!\!\_})_{\alpha} \text{Nothing} \text{Nothing} = \text{True}

(\mathit{=\!\!\!\!\_})_{\text{Maybe}} (\mathit{=\!\!\!\!\_})_{\alpha} (\text{Just} x) (\text{Just} x') = x \mathit{=\!\!\!\!\_} \alpha x'

(\mathit{=\!\!\!\!\_})_{\text{Maybe}} (\mathit{=\!\!\!\!\_})_{\alpha} \_ \_ = \text{False}
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Equality for parametrized datatypes

\textbf{data} Maybe \( \alpha = \) Nothing \mid Just \alpha

\((\implies)\) Maybe \( \implies \forall \alpha. (\alpha \to \alpha \to \text{Bool}) \to (\text{Maybe } \alpha \to \text{Maybe } \alpha \to \text{Bool})

\((\implies)\) Maybe \((\implies)\alpha\) Nothing Nothing \(=\) True

\((\implies)\) Maybe \((\implies)\alpha\) (Just \(x\)) (Just \(x'\)) \(=\) \(x \implies\alpha x'\)

\((\implies)\) Maybe \((\implies)\alpha\) \_ \_ \(=\) False

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Equality for parametrized datatypes

\[\text{data } \text{Maybe } \alpha = \text{Nothing} \mid \text{Just } \alpha\]

\[(\mathsf{==})_{\text{Maybe}} : \forall \alpha. (\alpha \to \alpha \to \text{Bool}) \to (\text{Maybe } \alpha \to \text{Maybe } \alpha \to \text{Bool})\]

\[(\mathsf{==})_{\text{Maybe}} (\mathsf{==})_{\alpha} \text{ Nothing Nothing} = \text{True}\]

\[(\mathsf{==})_{\text{Maybe}} (\mathsf{==})_{\alpha} (\text{Just } x) (\text{Just } x') = x \mathsf{==}_{\alpha} x'\]

\[(\mathsf{==})_{\text{Maybe}} (\mathsf{==})_{\alpha} - - = \text{False}\]

► We can define the equality for parametrized datatypes, but for that, we must know the equality function(s) for the argument(s).

► The equality function depends on itself.
Equality isn’t parametrically polymorphic

- We know intuitively what it means for two Packages to be equal.
- We also know what it means for two Trees, Maybes or TimeInfos to be equal.
- However, it is impossible to give a parametrically polymorphic definition for equality:

  \[
  (\Rightarrow) :: \forall \alpha. \alpha \to \alpha \to \text{Bool}
  \]

  \[
  x \Rightarrow y = ???
  \]

- This is a consequence of the Parametricity Theorem (Reynolds 1983).
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  \quad x \equiv y = ???
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- This is a consequence of the Parametricity Theorem (Reynolds 1983).
Overloading or ad-hoc polymorphism

- We have seen that we can define specific equality functions for many datatypes, following the intuitive algorithm that two values are equal iff
  - both values belong to the same alternative,
  - the corresponding fields are equal.

- A parametrically polymorphic equality function is impossible, because equality needs to access the structure of the datatypes to perform the comparison.

- Haskell allows to place functions that work on different types into a type class.

- Then, we can use the same name (==) for all the specific equality functions.
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- Haskell allows to place functions that work on different types into a type class.

- Then, we can use the same name (==) for all the specific equality functions.
A type class defines a set of datatypes that support common operations:

\[
\text{class } \text{Eq } \alpha \quad \text{where } (\equiv) :: \alpha \to \alpha \to \text{Bool}
\]

A type can be made an instance of the class by defining the class operations:

\[
\begin{align*}
\text{instance } \text{Eq } \text{TimeInfo} & \quad \text{where } (\equiv) = (\equiv)_{\text{TimeInfo}} \\
\text{instance } \text{Eq } \text{Package} & \quad \text{where } (\equiv) = (\equiv)_{\text{Package}} \\
\text{instance } \text{Eq } \alpha \Rightarrow \text{Eq [} \alpha \text{]} & \quad \text{where } (\equiv) = (\equiv)[\ ] (\equiv)
\end{align*}
\]

The dependency of equality turns into an instance constraint.
Type classes

A type class defines a set of datatypes that support common operations:

```haskell
class Eq α where (==) :: α → α → Bool
```

A type can be made an instance of the class by defining the class operations:

```haskell
instance Eq TimeInfo where (==) = (==) TimeInfo
instance Eq Package where (==) = (==) Package
instance Eq α ⇒ Eq [α] where (==) = (==) [ ] (==)
```

The dependency of equality turns into an instance constraint.
A type class defines a set of datatypes that support common operations:

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class Eq α where (==) :: α → α → Bool
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A type can be made an `instance` of the class by defining the class operations:

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instance Eq TimeInfo where (==) = (=)TimeInfo
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instance Eq α ⇒ Eq [α] where (==) = (=)[[]](==)
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The dependency of equality turns into an `instance constraint`.
Type classes

A type class defines a set of datatypes that support common operations:

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class Eq α where (==) :: α → α → Bool
```

A type can be made an instance of the class by defining the class operations:

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instance Eq TimeInfo where (==) :: TimeInfo → TimeInfo
instance Eq Package where (==) :: Package → Package
instance Eq α ⇒ Eq [α] where (==) [] = (==) [] (==) α
```

The dependency of equality turns into an instance constraint.
Is this satisfactory?

- We can use an overloaded version of equality on several datatypes now.
- We had to define all the instances ourselves, in an ad-hoc way.
- Once we want to use equality on more datatypes, we have to define new instances.

Let us define the equality function once and for all!
Is this satisfactory?

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Let us define the equality function once and for all!
Structural polymorphism (also called generic programming) makes the structure of datatypes available for the definition of type-indexed functions!
Generic programming in context

Ad-hoc polymorphism $\approx$ overloading

Parametric polymorphism
Generic programming in context

Ad-hoc polymorphism $\approx$ overloading

Structural polymorphism $\approx$ genericity

Parametric polymorphism
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Generic equality

\[(=) \langle \alpha \rangle \quad \text{::}\quad \alpha \to \alpha \to \text{Bool}\]

\[
\begin{align*}
(=) \langle \text{Unit} \rangle & \quad \text{Unit} \quad \text{Unit} \quad = \quad \text{True} \\
(=) \langle \text{Sum} \alpha \beta \rangle (\text{Inl} \; x) \quad (\text{Inl} \; x') & \quad = \quad (=) \langle \alpha \rangle \; x \; x' \\
(=) \langle \text{Sum} \alpha \beta \rangle (\text{Inr} \; y) \quad (\text{Inr} \; y') & \quad = \quad (=) \langle \beta \rangle \; y \; y' \\
(=) \langle \text{Sum} \alpha \beta \rangle \_ \quad \_ & \quad = \quad \text{False} \\
(=) \langle \text{Prod} \alpha \beta \rangle (x \times y) \quad (x' \times y') & \quad = \quad (=) \langle \alpha \rangle \; x \; x' \land (=) \langle \beta \rangle \; y \; y' \\
(=) \langle \text{Int} \rangle & \quad x \quad x' \quad = \quad (=) \text{Int} \; x \; x' \\
(=) \langle \text{Char} \rangle & \quad x \quad x' \quad = \quad (=) \text{Char} \; x \; x' \\
\end{align*}
\]

\[
\text{data} \quad \text{Unit} \quad = \quad \text{Unit} \\
\text{data} \quad \text{Sum} \alpha \beta \quad = \quad \text{Inl} \; \alpha \mid \text{Inr} \; \beta \\
\text{data} \quad \text{Prod} \alpha \beta \quad = \quad \alpha \times \beta
\]
Generic equality

\[
(=) \langle \alpha \rangle \quad \text{::} \quad \alpha \to \alpha \to \text{Bool}
\]

\[
(=) \langle \text{Unit} \rangle \quad \text{Unit} \quad \text{Unit} \quad = \quad \text{True}
\]

\[
(=) \langle \text{Sum } \alpha \beta \rangle \quad (\text{Inl } x) \quad (\text{Inl } x') \quad = \quad (=) \langle \alpha \rangle \quad x \quad x'
\]

\[
(=) \langle \text{Sum } \alpha \beta \rangle \quad (\text{Inr } y) \quad (\text{Inr } y') \quad = \quad (=) \langle \beta \rangle \quad y \quad y'
\]

\[
(=) \langle \text{Sum } \alpha \beta \rangle \quad \_ \quad \_ \quad = \quad \text{False}
\]

\[
(=) \langle \text{Prod } \alpha \beta \rangle \quad (x \times y) \quad (x' \times y') \quad = \quad (=) \langle \alpha \rangle \quad x \quad x' \quad \land \quad (=) \langle \beta \rangle \quad y \quad y'
\]

\[
(=) \langle \text{Int} \rangle \quad x \quad x' \quad = \quad (=) \langle \text{Int} \rangle \quad x \quad x'
\]

\[
(=) \langle \text{Char} \rangle \quad x \quad x' \quad = \quad (=) \langle \text{Char} \rangle \quad x \quad x'
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(=) \langle \alpha \rangle & \quad \forall \alpha \to \alpha \to \text{Bool} \\
(=) \langle \text{Unit} \rangle & \quad \text{Unit \ Unit} \quad = \text{True} \\
(=) \langle \text{Sum} \ \alpha \ \beta \rangle & \quad (\text{Inl} \ x) \ (\text{Inl} \ x') \quad = (=) \langle \alpha \rangle \ x \ x' \\
(=) \langle \text{Sum} \ \alpha \ \beta \rangle & \quad (\text{Inr} \ y) \ (\text{Inr} \ y') \quad = (=) \langle \beta \rangle \ y \ y' \\
(=) \langle \text{Sum} \ \alpha \ \beta \rangle & \quad - \ - \quad = \text{False} \\
(=) \langle \text{Prod} \ \alpha \ \beta \rangle & \quad (x \times y) \ (x' \times y') \quad = (=) \langle \alpha \rangle \ x \ x' \ \land \ (=) \langle \beta \rangle \ y \ y' \\
(=) \langle \text{Int} \rangle & \quad x \ x' \quad = (=) \text{Int} \ x \ x' \\
(=) \langle \text{Char} \rangle & \quad x \ x' \quad = (=) \text{Char} \ x \ x'
\end{align*}
\]

\textbf{data Unit} \quad = \text{Unit} \\
\textbf{data Sum} \ \alpha \ \beta = \text{Inl} \ \alpha \ \mid \ \text{Inr} \ \beta \\
\textbf{data Prod} \ \alpha \ \beta = \alpha \times \beta
Generic equality

\[(=) \langle \alpha \rangle \quad \quad :: \quad \alpha \to \alpha \to \text{Bool} \]

\[(=) \langle \text{Unit} \rangle \quad \text{Unit} \quad \text{Unit} \quad = \quad \text{True} \]

\[(=) \langle \text{Sum} \ \alpha \ \beta \ \rangle \quad (\text{Inl} \ x) \quad (\text{Inl} \ x') \quad = \quad (=) \langle \alpha \rangle \quad x \quad x' \]

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\[(=) \langle \text{Sum} \ \alpha \ \beta \ \rangle \quad \_ \quad \_ \quad = \quad \text{False} \]

\[(=) \langle \text{Prod} \ \alpha \ \beta \ \rangle \quad (x \times y) \quad (x' \times y') \quad = \quad (=) \langle \alpha \rangle \quad x \quad x' \quad \land \quad (=) \langle \beta \rangle \quad y \quad y' \]

\[(=) \langle \text{Int} \rangle \quad x \quad x' \quad = \quad (=) \langle \text{Int} \rangle \quad x \quad x' \]

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\textbf{data} \quad \text{Unit} \quad = \quad \text{Unit} \]

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Generic equality

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\textbf{data} \quad \text{Unit} \quad = \quad \text{Unit}

\textbf{data} \quad \text{Sum} \alpha \beta = \text{Inl} \alpha \mid \text{Inr} \beta

\textbf{data} \quad \text{Prod} \alpha \beta = \alpha \times \beta
Generic functions

A function that is defined for the Unit, Sum, and Prod types is “generic” or structurally polymorphic.

▶ It works automatically for “all” datatypes.
▶ Datatypes are implicitly deconstructed into a representation that involves Unit, Sum, and Prod.
▶ Primitive or abstract types might require special cases in the definition.
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Primitive types

- A primitive type is a datatype that can not be deconstructed because its implementation is hidden or because it cannot be defined by means of the Haskell `data` construct (such as `Int`, `Char`, `(→)`, and `IO`).

- If a generic function is supposed to work for types containing a primitive type, it has to be defined for this primitive type.

\[
(\equiv) \langle \text{Int} \rangle \; x \; x' = (\equiv)_{\text{Int}} \; x \; x' \\
(\equiv) \langle \text{Char} \rangle \; x \; x' = (\equiv)_{\text{Char}} \; x \; x'
\]

- Abstract types, where the programmer specifically hides the implementation, are treated in the same way as primitive types.
A **primitive** type is a datatype that can not be deconstructed because its implementation is hidden or because it cannot be defined by means of the Haskell `data` construct (such as `Int`, `Char`, `(→)`, and `IO`).

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\[
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\]
\[
(=) \langle \text{Char} \rangle \ x \ x' = (=)_{\text{Char}} \ x \ x'
\]

**Abstract types**, where the programmer specifically hides the implementation, are treated in the same way as primitive types.
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Abstract types, where the programmer specifically hides the implementation, are treated in the same way as primitive types.
Deconstruction into Unit, Sum, Prod

- A value of Unit type represents a constructor with no fields (such as Nothing or the empty list).
- A Sum represents the choice between two alternatives.
- A Prod represents the sequence of two fields.

```haskell
data Unit = Unit
data Sum α β = Inl α | Inr β
data Prod α β = α × β
```

```haskell
data Tree α = Leaf α | Node (Tree α) (Tree α)
Tree α ≈ Sum α (Prod (Tree α) (Tree α))
```
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Tree α ≈ Sum α (Prod (Tree α) (Tree α))
```
Using a generic function

The defined equality function can now be used at different datatypes.

```
data TimInfo = AM | PM | H24
data Tree α  = Leaf α | Node (Tree α) (Tree α)
```

```
(==) ⟨TimInfo⟩ AM H24        ~> False
(==) ⟨TimInfo⟩ PM PM         ~> True
(==) ⟨Tree Int⟩ (Node (Node (Leaf 2) (Leaf 4))
                  (Node (Leaf 1) (Leaf 3)))
                  (Node (Node (Leaf 4) (Leaf 2))
                  (Node (Leaf 1) (Leaf 3)))
       ~> False
```
Applications for generic functions

- comparison
  - equality
  - ordering

- parsing and printing
  - read/write a canonical representation
  - read/write a binary representation
  - read/write XML to/from a typed Haskell value
  - compression, encryption

- generation
  - generating default values
  - enumerating all values of a datatype
  - (random) generation of test data

- traversals
  - collecting and combining data from a tree
  - modifying data in a tree
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Advantages of generic functions

- reusable
- type safe
- simple
- adaptable
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Overview

About Haskell

Genericity and other types of polymorphism

Examples of generic functions

Generic Haskell
The Generic Haskell Project

- A research project funded by the NWO (Dutch Research Organisation) from October 2000 until October 2004.
- Goal: create a language extension for Haskell that supports generic programming.
- Based on earlier work by Johan Jeuring and Ralf Hinze.
- Project is now finished, but work on Generic Haskell will continue in Utrecht.
- Results: compared to the original ideas, much easier to use, yet more expressive.
- The PhD thesis “Exploring Generic Haskell” is a reasonably complete documentation of the results of the project.
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The Generic Haskell Compiler

- ... is a preprocessor for the Haskell language.
- It extends Haskell with constructs to define
  - type-indexed functions (which can be generic),
  - type-indexed datatypes.
- Generic Haskell compiles datatypes of the input language to isomorphic structural representations using Unit, Sum, and Prod.
- Generic Haskell compiles generic functions to specialized functions that work for specific types.
- Generic Haskell compiles calls to generic functions into calls to the specialisations.
Additional features

- Several mechanisms to define new generic definitions out of existing ones:
  - **local redefinition** allows to change the behaviour of a generic function on a specific type locally
  - **generic abstraction** allows to define generic functions in terms of other generic functions without fixing the type argument
  - **default cases** allow to extend generic functions with additional cases for specific types
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Several mechanisms to define new generic definitions out of existing ones:

- **Local redefinition** allows to change the behaviour of a generic function on a specific type locally
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- **Default cases** allow to extend generic functions with additional cases for specific types
Dependencies

- Generic functions can interact, i.e., depend on one another.
- For instance, equality depends on itself.
- There are generic functions that depend on multiple other generic functions.
- Dependencies are tracked by the type system in Generic Haskell.
Generic functions are functions defined on the structure of datatypes.

Type-indexed datatypes are datatypes defined on the structure of datatypes.

Type-indexed tries are finite maps that employ the shape of the key datatype to store the values more efficiently.

The zipper is a data structure that facilitates editing operations on a datatype.
Type-indexed datatypes

- Generic functions are **functions** defined on the structure of datatypes.
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- **Type-indexed tries** are finite maps that employ the shape of the key datatype to store the values more efficiently.
- The **zipper** is a data structure that facilitates editing operations on a datatype.
Implementation of Generic Haskell

- Generic Haskell can be obtained from www.generic-haskell.org.
- The current release (from January 2005) should contain all the features mentioned in this talk (except for syntactical differences when using type-indexed types).
Related work

- Scrap your boilerplate (Lämmel and Peyton Jones)
- Pattern calculus (Jay)
- Dependently typed programming (Augustsson, Altenkirch and McBride, …)
- Intensional type analysis (Harper, Morrisett, Weirich)
- GADTs (Cheney and Hinze, Weirich and Peyton Jones)
- Template Haskell (Sheard and Peyton Jones)
- Templates in C++
- Generics in C# and Java
- …
Future work

- Generic views, i.e., different structural representations of datatypes for different sorts of applications.
- Type inference.
- First-class generic functions.
- ...

Universiteit Utrecht
Many forms of parsing and printing functions can be written generically. A very simple example is a function to encode a value as a list of Bits:

```haskell
data Bit = O | I

encode ⟨α⟩ :: α → [Bit]
encode ⟨Unit⟩ Unit = []
encode ⟨Sum α β⟩ (Inl x) = O : encode ⟨α⟩ x
encode ⟨Sum α β⟩ (Inr y) = I : encode ⟨β⟩ y
encode ⟨Prod α β⟩ (x × y) = encode ⟨α⟩ x ++ encode ⟨β⟩ y
encode ⟨Int⟩ x = encodeInBits 32 x
encode ⟨Char⟩ x = encodeInBits 8 (ord x)
```
Parsing and printing – contd.

\textbf{data} \ Tree \ \alpha \ = \ \textit{Leaf} \ \alpha \ \mid \ \textit{Node} \ (\Tree \ \alpha) \ (\Tree \ \alpha) \\
\textbf{data} \ \text{TimInfo} \ = \ \textit{AM} \ \mid \ \textit{PM} \ \mid \ \textit{H24}

\textit{encode} \ \langle \text{TimInfo} \rangle \quad \text{H24} \\
\quad \leadsto [I,I] \\
\textit{encode} \ \langle \text{Tree TimInfo} \rangle \ (\text{Node} \ (\textit{Leaf} \ \textit{AM}) \ (\textit{Leaf} \ \textit{PM})) \\
\quad \leadsto [I,O,O,O,I,O]
Traversals

\[
\begin{align*}
\text{collect } \langle \alpha \rangle &:: \forall \rho. \alpha \to [\rho] \\
\text{collect } \langle \text{Unit} \rangle &\quad \text{Unit} \quad = [] \\
\text{collect } \langle \text{Sum } \alpha \beta \rangle (\text{Inl } x) &= \text{collect } \langle \alpha \rangle x \\
\text{collect } \langle \text{Sum } \alpha \beta \rangle (\text{Inr } y) &= \text{collect } \langle \beta \rangle y \\
\text{collect } \langle \text{Prod } \alpha \beta \rangle (x \times y) &= \text{collect } \langle \alpha \rangle x + \text{collect } \langle \beta \rangle y \\
\text{collect } \langle \text{Int} \rangle &\quad x \quad = [] \\
\text{collect } \langle \text{Char} \rangle &\quad x \quad = []
\end{align*}
\]

Alone, this generic function is completely useless! It always returns the empty list.
Traversals

\[
\begin{align*}
\text{collect } \langle \alpha \rangle & \quad :: \quad \forall \rho. \alpha \rightarrow [\rho] \\
\text{collect } \langle \text{Unit} \rangle & \quad \text{Unit} \quad = \quad [] \\
\text{collect } \langle \text{Sum } \alpha \beta \rangle & \quad (\text{Inl } x) \quad = \quad \text{collect } \langle \alpha \rangle \ x \\
\text{collect } \langle \text{Sum } \alpha \beta \rangle & \quad (\text{Inr } y) \quad = \quad \text{collect } \langle \beta \rangle \ y \\
\text{collect } \langle \text{Prod } \alpha \beta \rangle & \quad (x \times y) \quad = \quad \text{collect } \langle \alpha \rangle \ x \ + \ + \ \text{collect } \langle \beta \rangle \ y \\
\text{collect } \langle \text{Int} \rangle & \quad x \quad = \quad [] \\
\text{collect } \langle \text{Char} \rangle & \quad x \quad = \quad []
\end{align*}
\]

Alone, this generic function is completely useless! It always returns the empty list.
Local redefinition and \textit{collect}

The function \textit{collect} is a good basis for \textit{local redefinition}.

Collect all elements from a tree:

\begin{verbatim}
let collect ⟨τ⟩ x = [x]
in  collect ⟨Tree τ⟩ (Node (Leaf 1) (Leaf 2) (Leaf 3) (Leaf 4)) ↝ [1, 2, 3, 4]
\end{verbatim}
Local redefinition

\[
\text{let } (\equiv) \langle \alpha \rangle x y = (\equiv) \langle \text{Char} \rangle (\text{toUpper} \ x) (\text{toUpper} \ y) \\
\text{in } (\equiv) \langle [\alpha] \rangle "\text{generic Haskell}" "\text{Generic HASKELL}"
\]
Generic abstraction

\[
\text{symmetric } \langle \alpha \rangle x = \text{equal } \langle \alpha \rangle x (\text{reverse } \langle \alpha \rangle x)
\]
Type-indexed tries

type FMap ⟨Unit⟩ val = Maybe val
type FMap ⟨Sum α β⟩ val = (FMap ⟨α⟩ val, FMap ⟨β⟩ val)
type FMap ⟨Prod α β⟩ val = FMap ⟨α⟩ (FMap ⟨β⟩ val)