

# Datatype-generic Programming in Haskell

## An introduction

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Haven't you ever wondered  
how **deriving** works?

# Equality on binary trees

```
data T = L | N T T
```

Let's try ourselves:

# Equality on binary trees

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data T = L | N T T
```

Let's try ourselves:

```
eqT :: T -> T -> Bool
```

```
eqT L      L      = True
```

```
eqT (N x1 y1) (N x2 y2) = eqT x1 x2 ^ eqT y1 y2
```

```
eqT _      _      = False
```

# Equality on binary trees

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data T = L | N T T
```

Let's try ourselves:

```
eqT :: T → T → Bool
```

```
eqT L      L      = True
```

```
eqT (N x1 y1) (N x2 y2) = eqT x1 x2 ∧ eqT y1 y2
```

```
eqT _      _      = False
```

Easy enough, let's try another ...

## Equality on another type

```
data Choice = I Int | C Char | B Choice Bool | S Choice
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```
eqChoice :: Choice → Choice → Bool
```

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eqChoice (I n1 ) (I n2 ) = eqInt n1 n2
```

```
eqChoice (C c1 ) (C c2 ) = eqChar c1 c2
```

```
eqChoice (B x1 b1) (B x2 b2) = eqChoice x1 x2 ∧ eqBool b1 b2
```

```
eqChoice _ _ = False
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```

```
eqChoice (B x1 b1) (B x2 b2) = eqChoice x1 x2 ∧ eqBool b1 b2
```

```
eqChoice _ _ = False
```

Do you see a pattern?



## A pattern for defining equality

- ▶ How many cases does the function definition have?
- ▶ What is on the right hand sides?

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# A pattern for defining equality

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- ▶ What is on the right hand sides?
- ▶ How many clauses are there in the conjunctions on each right hand side?

Relevant concepts:

- ▶ number of constructors in datatype,
- ▶ number of fields per constructor,
- ▶ recursion leads to recursion,
- ▶ other types lead to invocation of equality on those types.

## More datatypes

```
data Tree a = Leaf a | Node (Tree a) (Tree a)
```

Like before, but with labels in the leaves.

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data Tree a = Leaf a | Node (Tree a) (Tree a)
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Like before, but with labels in the leaves.

How to define equality now?

We need equality on `a` !

```
eqTree :: (a → a → Bool) → Tree a → Tree a → Bool
eqTree eqa (Leaf n1 ) (Leaf n2 ) = eqa n1 n2
eqTree eqa (Node x1 y1) (Node x2 y2) = eqTree eqa x1 x2 ∧
                                             eqTree eqa y1 y2
eqTree eqa _ _ = False
```

# Type classes

Note how the definition of `eqTree` is perfectly suited for a type class instance:

```
instance Eq a  $\Rightarrow$  Eq (Tree a) where  
  (==) = eqTree (==)
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Note how the definition of `eqTree` is perfectly suited for a type class instance:

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```

In fact, type classes are usually implemented as **dictionaries**, and an instance declaration is translated into a **dictionary transformer**.



## Yet another equality function

This is often called a **rose tree**:

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data Rose a = Fork a [Rose a]
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Let's assume we already have:

```
eqList :: (a → a → Bool) → [a] → [a] → Bool
```

How to define `eqRose` ?

# Yet another equality function

This is often called a **rose tree**:

```
data Rose a = Fork a [Rose a]
```

Let's assume we already have:

```
eqList :: (a → a → Bool) → [a] → [a] → Bool
```

How to define `eqRose` ?

```
eqRose :: (a → a → Bool) → Rose a → Rose a → Bool  
eqRose eqa (Fork x1 xs1) (Fork x2 xs2) =  
    eqa x1 x2 ∧ eqList (eqRose eqa) xs1 xs2
```

No fallback case needed because there is only one constructor.

## More concepts

- ▶ Parameterization of types is reflected by parameterization of the functions.
- ▶ Application of parameterized types is reflected by application of the functions.

# The equality pattern

## An informal description

In order to define equality for a datatype:

- ▶ introduce a parameter for each parameter of the datatype,
- ▶ introduce a case for each constructor of the datatype,
- ▶ introduce a final catch-all case returning `False`,
- ▶ for each of the other cases, compare the constructor fields pair-wise and combine them using `( $\wedge$ )`,
- ▶ for each field, use the appropriate equality function; combine equality functions and use the parameter functions as needed.

# The equality pattern

## An informal description

In order to define equality for a datatype:

- ▶ introduce a parameter for each parameter of the datatype,
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- ▶ introduce a final catch-all case returning `False`,
- ▶ for each of the other cases, compare the constructor fields pair-wise and combine them using `(^)`,
- ▶ for each field, use the appropriate equality function; combine equality functions and use the parameter functions as needed.

If we can describe it, **can we write a program to do it?**

# Interlude: type isomorphisms

# Isomorphism between types

Two types `A` and `B` are called **isomorphic** if we have functions

$$f :: A \rightarrow B$$
$$g :: B \rightarrow A$$

that are mutual **inverses**, i.e., if

$$f \circ g \equiv \text{id}$$
$$g \circ f \equiv \text{id}$$



# Example

Lists and Snoc-lists are isomorphic

```
data SnocList a = Lin | SnocList a :> a
```

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## Lists and Snoc-lists are isomorphic

```
data SnocList a = Lin | SnocList a :> a
```

```
listToSnocList :: [a] → SnocList a
```

```
listToSnocList [] = Lin
```

```
listToSnocList (x : xs) = listToSnocList xs :> x
```

```
snocListToList :: SnocList a → [a]
```

```
snocListToList Lin = []
```

```
snocListToList (xs :> x) = x : snocListToList xs
```

We can prove that these are inverses.

## The idea of datatype-generic programming

If we can represent a type as an isomorphic type that is composed out of a limited number of type constructors, then we can define a function on each of the type constructors and gain a function that works on the original type – and in fact on any representable type.

## The idea of datatype-generic programming

If we can represent a type as an isomorphic type that is composed out of a limited number of type constructors, then we can define a function on each of the type constructors and gain a function that works on the original type – and in fact on any representable type.

In fact, we do not even quite need an isomorphic type.

For a type  $A$ , we need a type  $B$  and  $\text{from} :: A \rightarrow B$  and  $\text{to} :: B \rightarrow A$  such that

$$\text{to} \circ \text{from} \equiv \text{id}$$

We call such a combination an **embedding-projection pair**.

# Choice between constructors

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```
data Either a b = Left a | Right a
```



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Which type best encodes choice between constructors?

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Booleans encode choice, but do not provide information what the choice is about.

```
data Either a b = Left a | Right a
```

Choice between three things:

```
type Either3 a b c = Either a (Either b c)
```

# Combining constructor fields

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```

Combining three fields:

```
type Triple a b c = (a, (b, c))
```

# What about constructors without arguments?

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Well, how many values does a constructor without argument encode?

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Well, how many values does a constructor without argument encode?

```
data () = ()
```



# Representing types

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To keep representation and original types apart, let's define isomorphic copies of the types we need:

```
data U      = U  
data a :+: b = L a | R b  
data a **: b = a **: b
```

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We can now get started:

```
data Bool = False | True
```

How do we represent `Bool` ?

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data a **: b = a **: b
```

We can now get started:

```
data Bool = False | True
```

How do we represent `Bool` ?

```
type RepBool = U :+: U
```

# A class for representable types

```
class Representable a where  
  type Rep a  
  from :: a → Rep a  
  to   :: Rep a → a
```

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```
class Representable a where  
  type Rep a  
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  to   :: Rep a → a
```

The type `Rep` is an **associated type**. GHC allows us to define datatypes and type synonyms within classes, depending on the class parameter(s).

# Representable Booleans

```
instance Representable Bool where  
  type Rep Bool = U :+: U  
  from False = L U  
  from True  = R U  
  to (L U) = False  
  to (R U) = True
```

# Representable Booleans

```
instance Representable Bool where  
  type Rep Bool = U :+: U  
  from False = L U  
  from True  = R U  
  to (L U) = False  
  to (R U) = True
```

## Question

Are Bool and Rep Bool isomorphic?



# Representable lists

```
instance Representable [a] where  
  type Rep [a] = U :+: (a :+: [a])  
  from []           = L U  
  from (x : xs)    = R (x :+: xs)  
  to (L U          ) = []  
  to (R (x :+: xs)) = x : xs
```

# Representable lists

```
instance Representable [a] where  
  type Rep [a] = U :+: (a :+: [a])  
  from []           = L U  
  from (x : xs)    = R (x :+: xs)  
  to   (L U       ) = []  
  to   (R (x :+: xs)) = x : xs
```

Note that the representation of recursive types mentions the original types – if needed, we can apply the transformation multiple times.

# Representable lists

```
instance Representable [a] where  
  type Rep [a] = U :+: (a :+: [a])  
  from []           = L U  
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  to (L U          ) = []  
  to (R (x :+: xs)) = x : xs
```

Note that the representation of recursive types mentions the original types – if needed, we can apply the transformation multiple times.

Note further that we do not require `Representable a`.

# Representable trees

```
instance Representable (Tree a) where  
  type Rep (Tree a) = a :+: (Tree a **: Tree a)  
  from (Leaf n      ) = L n  
  from (Node x y    ) = R (x **: y)  
  to   (L n         ) = Leaf n  
  to   (R (x **: y)) = Node x y
```

# Representable rose trees

```
instance Representable (Rose a) where  
  type Rep (Rose a) = a :: [Rose a]  
  from (Fork x xs) = x :: xs  
  to   (x :: xs ) = Fork x xs
```

# Representing primitive types

For some types, it does not make sense to define a structural representation – for such types, we will have to define generic functions directly.

```
instance Representable Int where  
  type Rep Int = Int  
  from = id  
  to   = id
```

# Back to equality

## Intermediate summary

- ▶ We have defined class `Representable` that maps datatypes to representations built up from `U`, `(:+:)`, `(:*:)` and other datatypes.
- ▶ If we can define equality on the representation types, then we should be able to obtain a generic equality function.
- ▶ Let us apply the informal recipe from earlier.



## Equality on sums

```
eqSum :: ( a      → a      → Bool) →  
         (      b →      b → Bool) →  
         a :+: b → a :+: b → Bool
```

```
eqSum eqa eqb (L a1) (L a2) = eqa a1 a2
```

```
eqSum eqa eqb (R a1) (R a2) = eqb a1 a2
```

```
eqSum eqa eqb _      _      = False
```

# Equality on products

```
eqProd :: ( a      → a      → Bool) →  
          (      b →      b → Bool) →  
          a :: b → a :: b → Bool
```

```
eqProd eqa eqb (a1 :: b1) (a2 :: b2) =  
  eqa a1 a2 ∧ eqb b1 b2
```

# Equality on units

```
eqUnit :: U → U → Bool  
eqUnit U U = True
```

What now?

## A class for generic equality

```
class GEq a where  
  geq :: a → a → Bool
```

# A class for generic equality

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class GEq a where  
  geq :: a → a → Bool
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```
instance (GEq a, GEq b) ⇒ GEq (a :+: b) where  
  geq = eqSum geq geq
```

```
instance (GEq a, GEq b) ⇒ GEq (a **: b) where  
  geq = eqProd geq geq
```

```
instance GEq U where  
  geq = eqUnit
```

## A class for generic equality

```
class GEq a where  
  geq :: a → a → Bool
```

```
instance (GEq a, GEq b) ⇒ GEq (a :+: b) where  
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```

```
instance (GEq a, GEq b) ⇒ GEq (a **: b) where  
  geq = eqProd geq geq
```

```
instance GEq U where  
  geq = eqUnit
```

Instances for primitive types:

```
instance GEq Int where  
  geq = eqInt
```

## Dispatching to the representation type

```
eq :: (Representable a, GEq (Rep a)) => a -> a -> Bool
eq x y = geq (from x) (from y)
```



## Dispatching to the representation type

```
eq :: (Representable a, GEq (Rep a)) => a -> a -> Bool
eq x y = geq (from x) (from y)
```

Defining generic instances is now trivial:

```
instance          GEq Bool      where
  geq = eq
instance GEq a => GEq [a]      where
  geq = eq
instance GEq a => GEq (Tree a) where
  geq = eq
instance GEq a => GEq (Rose a) where
  geq = eq
```

Have we won  
or  
have we lost?

# Amount of work

## Question

Haven't we just replaced some tedious work (defining equality for a type) by some other tedious work (defining a representation for a type)?

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Haven't we just replaced some tedious work (defining equality for a type) by some other tedious work (defining a representation for a type)?

Yes, but:

- ▶ The representation has to be given only once, and works for potentially many generic functions.
- ▶ Since there is a single representation per type, it could be generated automatically by some other means (compiler support, TH).

# Other generic functions

# Coding and decoding

We want to define

```
data Bit = O | I
```

```
encode :: (Representable a, GEncode (Rep a)) => a -> [Bit]
```

```
decode :: (Representable a, GDecode (Rep a)) => BitParser a
```

```
type BitParser a = [Bit] -> Maybe (a, [Bit])
```

such that encoding and then decoding yields the original value.

# What about constructor names?

Seems that the representation we have does not provide constructor name info.

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So let us extend the representation:

```
data C c a = C a
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Note that `c` does not appear on the right hand side.



## What about constructor names?

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So let us extend the representation:

```
data C c a = C a
```

Note that `c` does not appear on the right hand side.

But `c` is supposed to be in this class:

```
class Constructor c where  
  conName :: t c a → String
```

## Trees with constructors

```
data TreeLeaf
```

```
instance Constructor TreeLeaf where
```

```
  conName _ = "Leaf"
```

```
data TreeNode
```

```
instance Constructor TreeNode where
```

```
  conName _ = "Node"
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# Trees with constructors

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```
instance Constructor TreeNode where
```

```
  conName _ = "Node"
```

```
instance Representable (Tree a) where
```

```
  type Rep (Tree a) = C TreeLeaf a :+:
```

```
                    C TreeNode (Tree a :+: Tree a)
```

```
  from (Leaf n          ) = L (C n)
```

```
  from (Node x y        ) = R (C (x :+: y))
```

```
  to   (L (C n)          ) = Leaf n
```

```
  to   (R (C (x :+: y))) = Node x y
```

## Defining functions on constructors

```
instance (GShow a, Constructor c) ⇒ GShow (C c a) where  
  gshow c@(C a)  
    | null args  = conName c  
    | otherwise = "(" ++ conName c ++ " " ++ args ++ "  
where args = gshow a
```

## Defining functions on constructors

```
instance (GShow a, Constructor c)  $\Rightarrow$  GShow (C c a) where  
  gshow c@(C a)  
    | null args  = conName c  
    | otherwise = "(" ++ conName c ++ " " ++ args ++ "  
where args = gshow a
```

```
instance (GEq a)  $\Rightarrow$  GEq (C c a) where  
  geq (C x) (C y) = geq x y
```

# A library for generic programming

What we have discussed so far is available on Hackage as a library called **instant-generics**.

- ▶ `Representable` instances for most prelude types.
- ▶ Template Haskell generation of `Representable` instances.
- ▶ A number of example generic functions.
- ▶ Additional markers in the representation to distinguish positions of type variables from other fields.

Is this the only way?

# Many design choices

**No!**

There are lots of approaches (too many) to generic programming in Haskell.



# Many design choices

**No!**

There are lots of approaches (too many) to generic programming in Haskell.

- ▶ The main question is exactly **how** we represent the datatypes – we have already seen what kind of freedom we have.
- ▶ The view dictates which datatypes we can represent easily, and which generic functions can be defined.

# Other notable approaches

## Constructor-based views

The **Scrap your boilerplate** library takes a very simple view on values:

```
C x1 ... xn
```

Every value in a datatype is a constructor applied to a number of arguments.

# Other notable approaches

## Constructor-based views

The **Scrap your boilerplate** library takes a very simple view on values:

```
C x1 ... xn
```

Every value in a datatype is a constructor applied to a number of arguments.

Using SYB, it is easy to define traversals and queries.

# Other notable approaches

## Children-based views

The **Uniplate** library is a simplification of SYB that just shows how in a recursive structure we can get to the children, and back from the children to the structure.

```
uniplate :: Uniplate a ⇒ a → ([a], [a] → a)
```

# Other notable approaches

## Children-based views

The **Uniplate** library is a simplification of SYB that just shows how in a recursive structure we can get to the children, and back from the children to the structure.

```
uniplate :: Uniplate a ⇒ a → ([a], [a] → a)
```

While a bit less powerful than SYB, this is one of the simplest Generic Programming libraries around, and allows to define the same kind of traversals and queries as SYB.

# Other notable approaches

## Fixed-point views

The **regular** and **multirec** libraries work with representations that abstract from the recursion by means of a fixed-point combinator, in addition to revealing the sums-of-product structure

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The **regular** and **multirec** libraries work with representations that abstract from the recursion by means of a fixed-point combinator, in addition to revealing the sums-of-product structure

```
data Fix f = In (f (Fix f))  
out (In f) = f
```

# Other notable approaches

## Fixed-point views

The **regular** and **multirec** libraries work with representations that abstract from the recursion by means of a fixed-point combinator, in addition to revealing the sums-of-product structure

```
data Fix f = In (f (Fix f))  
out (In f) = f
```

Using a fixed-point view, we can more easily capture functions that make use of the recursive structure of a type, such as folds and unfolds (catamorphisms and anamorphisms).



# GHC implementation

An approach that is quite similar to instant-generics has just been implemented directly in GHC, and will be available in the upcoming 7.2.1 release together with the Hackage library **generic-deriving**.

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An approach that is quite similar to instant-generics has just been implemented directly in GHC, and will be available in the upcoming 7.2.1 release together with the Hackage library **generic-deriving**.

With this approach, GHC can automatically (without using TH) generate the representations for you.

## Outlook: dependent types

Dependently typed programming languages such as **Agda** allow types to depend on terms. For example,

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Vec Int 5
```

could be a vector of integers of length `5`.

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Dependently typed programming languages such as **Agda** allow types to depend on terms. For example,

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Vec Int 5
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could be a vector of integers of length `5`.

We can also compute types from values, then. So we can define grammars of types as normal datatypes, and interpret them as the types they describe.

Makes it easy to play with many different views (universes).

## Other topics

There is more than we can cover in this lecture:

- ▶ Looking at all the other GP approaches closely.
- ▶ Comparison with template meta-programming.
- ▶ Efficiency of generic functions.
- ▶ Type-indexed types.
- ▶ ...

# Questions?